

On fluid/wall slippage

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Abstract

Certain (non polymeric) fluids show an anomalously low friction when flowing against well chosen solid walls. We discuss here one possible explanation, postulating that a gaseous film of small thickness h is present between fluid and wall. When h is smaller than the mean free path ℓ of the gas (Knudsen regime) the Navier length b is expected to be independent of h and very large (microns).

1 Introduction

The standard boundary condition for fluid flow along a wall is a no slip condition. If the wall is at rest, the tangential fluid velocity at the wall vanishes. The validity of this postulate was already checked in pioneering experiments by Coulomb [1].

However, the spatial resolution available to Coulomb was limited. In our days, we characterise the amount of slip by an extrapolation length b (the Navier length): the definition of b is explained on fig. 1. The length b can be related to the surface friction coefficient k defined as follows: the shear stress σ induces at the wall a surface velocity v_s :

$$\sigma = kv_s \quad (1)$$

Equating this to the viscous shear stress in the fluid (of velocity η), we get:

$$\sigma = kv_s = \eta \left| \frac{dv(z)}{dz} \right| \quad (2)$$

where $v(z)$ is the fluid velocity, increasing linearly with the distance from the wall z . When compared with fig. 1, we see that eq. 2 gives:

$$b = \frac{\eta}{k} \quad (3)$$

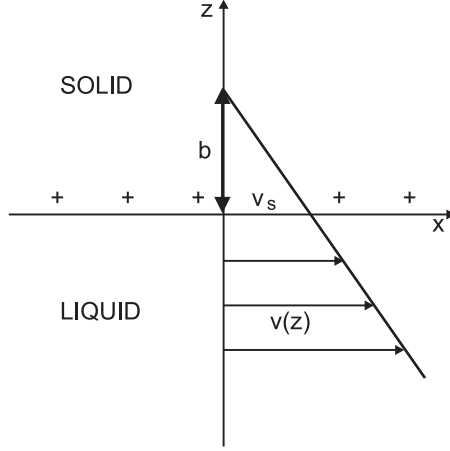


Figure 1: Definition of the Navier length b for simple shear flow near a solid wall

For most practical situations, with simple fluids (made of small molecules with a diameter a), we expect a very small Navier length $b \sim a$: this has been verified in classical experiments, using a force machine under slow shear [2].

1) A polymer melt, facing a carefully prepared surface, with grafted chains which are chemically identical to the melt: here, when σ becomes larger than a certain critical value σ^* , the Navier length jumps to high values (~ 50 microns) [3]. This can plausibly be explained in terms of polymer dynamics [4]: at $\sigma > \sigma^*$, the grafted chains are sufficiently elongated by the flow to disentangle from the moving chains.

2) With water flowing in thin, hydrophobic, capillaries, there is some early qualitative evidence for slippage [5], [6].

3) The role of the hydrophobic surfaces has been analysed theoretically by J. L. Barrat and coworkers -using analysis plus simulations [7], [8]. They show that the first layer of "water" molecules is depleted in the presence of a hydrophobic wall: this can lead in their case to $b \sim 15$ molecular diameters in a typical case, for a contact angle $\theta = 150^\circ$, corresponding to strong hydrophobicity.

4) Recent experiments by Hervet, Léger and coworkers are based on a local photobleaching technique (using evanescent waves) which probes the velocity field $v(z)$ within 50 nanometers of the surface [9]. They studied hexadecane flowing on sapphire: with bare sapphire, no slip was detected. On a fully grafted (methylated) sapphire, the slip length was found to be large (175 nm) and independent of σ in the observed range. This is a surprise: a) we do not expect any hydrophobic effect here b) even if it was present, the b values are much larger than the Barrat-Bocquet estimates.

5) Similar (as yet unpublished) results have been obtained by Zhu and Granick (using a force machine) and by Tabeling (using etched microcapillaries).

The results [4] and [5] are unexpected and stimulating. This led us to think about unusual processes which could take place near a wall. In the present note, we discuss one (remote) possibility: the formation of a gaseous film at the solid/liquid interface.

The source of the film is unclear: when the contact angle is large ($\theta \rightarrow 180^\circ$), a type of flat bubble can form at the surface with a relative low energy. But this energy is still high when compared to kT .

We first considered flat vapor bubbles generated at the solid fluid interface by thermal fluctuations. But the typical thickness of these bubbles turns out to be very small: of order $(kT/\gamma)^{1/2}$ (where T is the temperature, and γ the surface tension), even when θ is closed to 180° : thus, this brings us back to the Barrat picture, with depletion in one first layer of the liquid.

Another possible source is an external gas dissolved in the liquid, up to metastable concentrations: this would then nucleate bubbles preferentially near the wall if $\theta > 90^\circ$.

In section 2, we simply assume that there exists a uniform gas film of thickness h , larger than the molecule size a , but smaller than the mean free path ℓ in the gas. We calculate the Navier length b for this case, and find large values. The possible meaning of this result is discussed in section 3.

2 A gas film in the Knudsen regime

The situation is shown on fig. 2. In the Knudsen regime, gas molecules travel freely in the film and hit the boundaries. We assume that there is no specular reflection on either boundary. Then, a molecule leaving the liquid will have a gaussian velocity distribution for the tangential component v_x , with the peak of this gaussian at velocity v_s (the translational velocity at the liquid surface). On the average, it will transmit a momentum mv_s to the solid. The number of such hits per second is $\rho/m v_z$, where ρ is the gas density, m the molecular mass, and v_z the normal component of velocity.

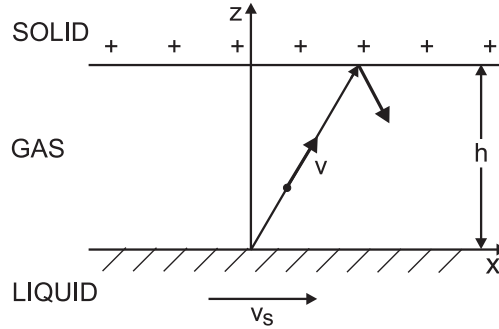


Figure 2: A gas molecule leaves the liquid with the velocity \vec{v} , and transmits (on the average) a momentum mV to the solid surface.

This leads to a shear stress:

$$\sigma = mV_s \frac{\rho}{m} \bar{v}_z = \rho V_s \bar{v}_z \quad (4)$$

with:

$$\bar{v}_z = \int_0^\infty \frac{1}{\sqrt{2\pi} v_{th}} v_z e^{-v_z^2/2v_{th}^2} dv_z = v_{th}/\sqrt{2\pi} \quad (5)$$

where $v_{th}^2 = kT/m$.

Eq. (4) gives us a precise value of the friction coefficient k in eq. (2), and from it, we arrive at a Navier length:

$$b = -h + \frac{\eta}{\rho \bar{v}_x} \cong \frac{\eta}{\rho v_z} \quad (h < \ell) \quad (6)$$

(The $-h$ term is due to the distance between liquid surface and solid surface; it is completely negligible in practice).

If we choose typical values, $\rho = 1\text{g/liter}$, $v_{th} = 300\text{m/sec}$, and $\eta = 10^{-2}\text{poises}$, we find $b = 7\text{ microns}$. Thus, a gas film can indeed give a very large slip length. Our calculation assumed complete thermalisation at each particle/boundary collision. If we had some non zero reflectance (especially on the solid surface), this would increase b even more.

3 Conclusions

When gas films with thickness h in the interval $a \ll h \ll \ell$ are present in a flow experiment, they may indeed generate a strong slippage, independent of the film thickness. But the process which could generate such films remains obscure. If, for some reason, the liquid entering the channel was supersaturated with a certain gas, a pressure drop in the channel could indeed promote the release of gas bubbles. If the solid surface is not very wettable ($\theta > 90^\circ$), the bubbles should preferentially nucleate at the liquid/solid interface (as they do in a glass of champagne). Then, we would have to postulate that the shear flow transforms them into a flat film, provided that the shear stress is strong enough to flatten them. Indeed, in the Zhu-Granick experiments, there is a threshold stress, but it is much smaller than what would result from a compromise between shear stress and surface tension for small bubbles.

It is worth emphasizing that the amount of gas required to lubricate the solid liquid contact is very small: if D is the macroscopic width of the shear flow, the weight fraction of gas required is:

$$\psi = \frac{\rho}{\rho_L} \frac{h}{D} \quad (7)$$

where ρ_L is the density of the liquid. Typically $\psi \sim 10^{-5}$.

On the whole, we cannot present a complete explanation of the anomalous Navier lengths based on gas films. But the films may possibly show up in some instances; e.g. if the liquid is purposely oversaturated with gas: then eq. 6 may become useful.

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